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A NOTE ON AN NP-COMPLETE PROBLEM, (U)  
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A Note on an NP-Complete Problem

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We show in this note that the problem of two-coloring a graph  $G = (V, E)$  with a distinguished vertex  $p$  and two distinguished subset  $A(p)$  and  $B(p)$  of  $V$ , such that there is a red connected subgraph containing  $A + p$ , and a blue connected subgraph containing  $B + p$ , is NP-Complete. We prove this by reduction of 3-satisfiability [Aho] by constructing, for a given boolean expression in disjunctive normal form with three variables in each term, a graph  $G$  with a distinguished vertex  $D$  and two subsets  $A(D)$  and  $B(D)$  such that the expression is satisfiable if and only if the graph has a two-coloring with the desired monochromatic subtrees.

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In constructing our graph, for a given expression  $E$  in variables  $x_1, x_2, \dots, x_n$  there will be two types of configurations. The first one corresponds to a variable (Figure 1). For each variable, we construct one of these configurations and connect it to what will be our distinguished vertex  $D$  by an edge from the top vertex. In this construction, we let the edge on the left side of the configuration correspond to the un-complemented form  $x_i$  of the  $i$ th variable  $x_i$  and the right side correspond to the complemented form  $x'_i$ . Now, for each term  $t_j$  in the expression, we construct a copy of the second configuration (Figure 2) and again connect the top vertex of the configuration to  $D$  by an edge. Then we identify the three edges in the middle of the configuration to three of the middle edges which are in the "variable" configurations in the following way: If the  $j$ th term contains the three variable forms  $y_{ia}, y_{ib}$ , and  $y_{ic}$ ; where, for example,  $y_{ia}$  is either the complemented or un-complemented form of some variable  $x_j$ , then we identify the edge  $y_{ia}$  to the corresponding edge in the  $x_j$  "variable" configuration. This implies that the two vertices connected to this edge are also identified everywhere the variable form  $y_{ia}$  appears.

So we have constructed a graph (Figure 3) which corresponds to the given expression. We have here drawn the edges in the two sides of the graph as disjoint for simplicity; but in the graph, we must remember that all of the edges which correspond to a particular form of a variable are identified. We note here that this construction is

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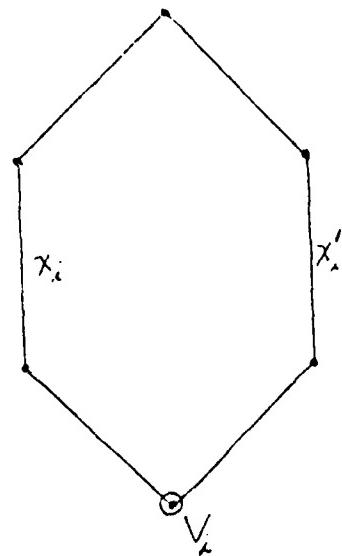


Figure 1 Variable

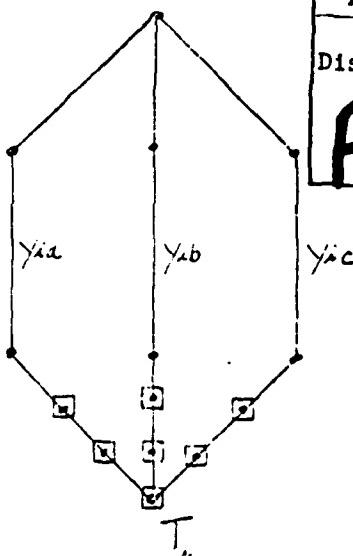


Figure 2 Term

clearly polynomial in time, (and also space). It should also be noted that our problem is clearly in NP, since if we are given an alleged solution, we can easily check to determine if all of the communication constraints are satisfied by performing a depth first search from D to see which vertices are reachable from it by blue and red paths, and then to check if the two distinguished subsets are included in the reachable vertices for the corresponding color.

We now form the two subsets of vertices in the following way: A (D) (or simply A) consists of all of the "round" vertices, and B (D) (or simply B) of the "square" ones. We note that none of these vertices is incident to an edge which has been identified.

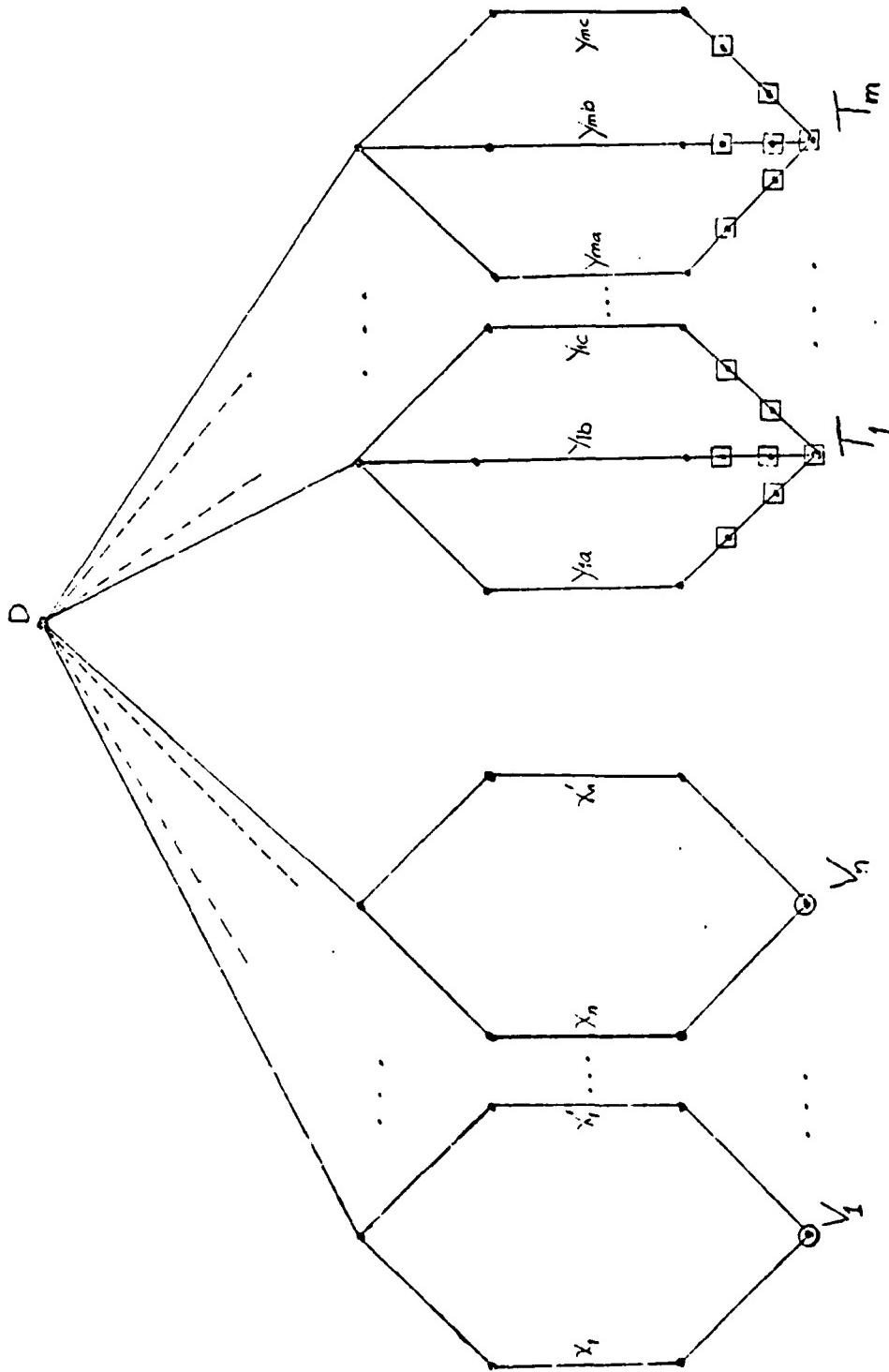


Figure 3. The Corresponding Graph

"variable" configuration (say  $x_i$ ) both of the edges are blue ("true") and if there is to be a red path from the "round" vertex  $V_i$  in the  $i$ th "variable" configuration to D, it must "sneak through" to D through the "expression" side of the graph. If we assume that it starts this "sneak path" through the term  $t_j$ , it must go through the vertex  $T_j$  since it can't go directly upwards because this edge is, by hypothesis, blue. This red path will isolate some "square" vertex by two red edges (and note that these are the only edges leading to this "square" vertex). However, this contradicts the fact that there is a blue path from this vertex to D. Therefore one of the edges in each "variable" configuration must be red.

It may be the case that both of these edges might be red, but this indicates that the variable has nothing to do with the truth value of the expression, it cannot appear in any blue path, and is a "don't care" which can have no effect on the satisfiability of the expression. This completes the proof to show that the graph has a suitable orientation if and only if the expression is satisfiable; and therefore, this edge-colorability problem is NP-Complete.